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Isoperiodic deformations of solutions of the Harry Dym hierarchy

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The Harry Dym (HD) hierarchy is a infinite set of completely integrable nonlinear PDEs that are related to the spectral theory of the acoustic operator, as well as certain physical problems, such as the geodesic flows on an N-dimensional ellipsoid and the theory of Hele-Shaw interface evolution. The set of its periodic solutions has in principle been constructed [1], however, the known formulae are ineffective. Knowledge of the periodic solutions of HD will allow us to describe all periodic finite-gap potentials of the string equation and all closed orbits of a particle confined to an ellipsoid in N dimensions.

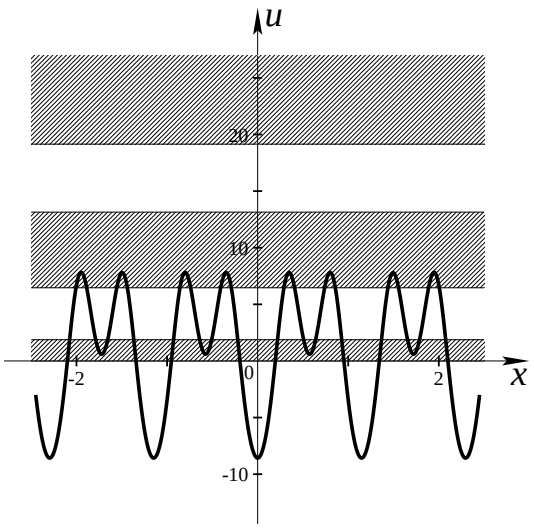
The HD hierarchy

$$r_t = r^3 \left(-\partial_x^3 r \partial_x^{-1} r\right)^m r^{-3} r_x \qquad (1)$$

can be integrated using a reciprocal link to an equation which reduces to the Korteweg-de Vries (KdV) hierarchy:

$$u_t = -\left(\partial_x^2 - 4u - 2u_x \partial_x^{-1}\right)^m u_x \qquad (2)$$

The periodic solutions of KdV have been known since 1974 [2]. However, the known formulae did not allow one to effectively distinguish periodic and quasiperiodic solutions. The set of all finite-gap periodic solutions can be effectively described using the recently discovered method of isoperiodic deformation [3]. This involves deforming the spectral data that defines the Riemann manifold on which we construct the solution, such that the periods of a certain meromorphic 1-form are preserved, which, in turn, preserves the periods of the corresponding solution. Together with Mikhail Stepanov, we have written a program that implements the deformations and explicitly constructs finite-gap solutions of KdV.



Two-gap solution of KdV obtained using isoperiodic deformation, for $t = 0$. The white bands are the forbidden gaps of the spectrum of the corresponding Schrödinger operator.

The inverse transformation from HD to KdV is implicit, therefore the periods of the obtained solutions are not preserved. My goal is to find a construction similar to isoperiodic deformation for the HD case and obtain a full description of its periodic solutions.

References

[1] L. A. Dmitrieva, Finite-gap solutions of the Harry Dym equation, Phys. Lett. A, 182 (1993), 65-70.

[2] S. P. Novikov, Theory of Solitons, Consultants Bureau, 1984.

[3] P. G. Grinevich, M. U. Schmidt, Period preserving nonisospectral flows and the moduli space of periodic solutions of soliton equations, Physica D, 87 (1995), 73-98.